Individuals and Their Guises: A Property-theoretic Analysis

Chris Fox

University of Essex and Universität des Saarlandes[1]

Abstract
This paper reappraises Landman's formal theory of *intensional individuals*—individuals under *roles*, or *guises* (Landman 1989)—within property theory (PT) (Turner 1992). As many of Landman's axioms exist to overcome the strong typing of his representation, casting his ideas in weakly typed PT produces a simpler theory. However, there is the possibility of an even greater simplification: if roles, or guises, are represented with *property modifiers* then there is no need for Landman's intensional individuals. Landman's argument against the use of property modifiers is re-examined, and shown to be mistaken.

1 Introduction

This paper is concerned with the problem of representing sentences involving reference to individuals under different guises: that is, how to allow the predication of different properties of the same individuals when referred to by different nominals. The following examples, which Landman seeks to address (Landman 1989), illustrate the problem:

> The judge is strict.
> John is liberal.
> John is the judge.

It seems that all three statements can be true without contradiction, even if 'strict' and 'liberal' are taken to be contradictory. Landman suggests that, in the representation, a contradiction can be avoided if the nominals are taken to refer to individuals acting under some *role*, or *guise*. According to this view the sentences give rise to the following interpretation:[2]

> John-as-a-judge is strict.
> John-as-himself is liberal.

Similarly, with the sentences:

> The cleaners are on strike.
> The judges are not on strike.

it may be the case that a person, John, is both a judge and a cleaner. Landman seeks to avoid any contradiction by taking the following to be the appropriate consequence:

> John-as-a-cleaner is on strike.
> John-as-a-judge is not on strike.

Landman gives a formal theory in which these *intensional individuals*—individuals under roles or guises—can be represented. He takes these objects to be of the same type as generalised quantifiers.

---

1. This paper was researched while I was at the University of Essex on a Science and Engineering Research Council Grant, and written while at the Universität des Saarlandes.
2. Note that these are taken to be paraphrases of the formalisation. It is not the intention of this paper to address the semantics of natural language "...as a ..." constructions.

A problem with Landman's formalism is that it is strongly typed. This creates complications for any theory of plural individuals. For this reason, he suggests that it might be useful to examine these ideas in a more weakly typed framework. This is one goal of the paper: I shall present a property theory PT, a weakly typed first-order theory, and show how Landman's ideas can be simplified by taking individuals under guises to be a new class of terms in PT.

It could be argued that PT is *too* weakly typed, as there is no constraint on term formation: we may formally keep iterating the construction of individuals under guises. It is not intuitively obvious what function, if any, such iterated constructions might have in natural language semantics. The absence of relevant intuitions could be reflected by adopting axioms that enforce no particular behaviour on such constructions. However, there is another possibility, which is to use *property modifiers* to represent these problematic sentences: roles or guises would then give rise to the modification of a property, rather than a modification of an individual. Landman would agree that you can often do this, but he presents examples involving comparatives which he thinks *must* be represented by individuals under guises. I reconsider the truth conditions of these examples, and show that they *can* be treated with property modifiers. As it is weakly typed, PT is a useful vehicle for a theory of property modifiers: a term can be taken to be both a property and a property modifier.

2 Landman's Formal Theory

Landman's formal theory is constructed within a strongly typed intensional logic, based upon Thomason's model of propositional attitudes (Thomason 1980). The intensional individuals form *quantifiers* rather than terms.

In Landman's formal theory, if t is a term and P is a predicate, then $t \varrho P$ is an expression denoting t restricted to its aspect of having P. The expression $t \varrho P$ is not a new term. Landman cannot deal with it in this way as, the logic he adopts has a poor language of terms.

Properties are of type $\langle e, p\rangle$. The denotation of NPs are second order properties of type $\langle\langle e, p\rangle, p\rangle$. John in all his aspects, $\lambda P.Pj'$, is such a second order property. John, himself, is of type e. John-as-a-P, is $j \varrho P$, where j is of type e, P of type $\langle e, p\rangle$ and $j \varrho P$ of type $\langle\langle e, p\rangle, p\rangle$. This is summarised in the following table:

Notion	Representation	Type
Properties	P	$\langle e, p\rangle$
Noun phrases	*Generalised quantifiers*	$\langle\langle e, p\rangle, p\rangle$
John, in all his aspects	$\lambda P.Pj$	$\langle\langle e, p\rangle, p\rangle$
John (himself)	j	e
John-as-a-P	$j \varrho P$	$\langle\langle e, p\rangle, p\rangle$

The expression:

$$(j \varrho \text{Judge}')(\text{Strike}')$$

represents the proposition that John, under the guise of being a judge, is on strike.

2.1 Landman's Axioms

The first three axioms concern the nature of individuals under roles:

(i)	John-as-a-judge is John:	$(j \varrho J)\lambda x(x = j)$.
(ii)	John-as-a-judge is a judge:	$(j \varrho J)(J)$.
(iii)	John-as-John is John:	$(j \varrho \lambda x(x = j)) = \lambda P(Pj)$.

The next four axioms give 'predication' of restricted individuals the expected behaviour:

(iv) $(j\,\varrho\,J)(P)\,\&\,(j\,\varrho\,J)(Q) \rightarrow (j\,\varrho\,J)\lambda x(Px\,\&\,Qx)$.
(v) $(j\,\varrho\,J)(P)\,\&\,P \vdash Q \rightarrow (j\,\varrho\,J)(Q)$.
(vi) $\sim\exists P((j\,\varrho\,J)(\lambda x(Px\,\&\,\sim Px)))$.
(vii) $\forall P((j\,\varrho\,J)(\lambda x(Px \vee \sim Px)))$.

Essentially: Axioms (iv), (v) make restricted terms into filters of properties. Axiom (vi) additionally makes them proper filters, and Axiom (vii), ultra-filters.

The final axiom seeks to constrain the appearance of restricted individuals:

(viii) $(j\,\varrho\,J)(P) \rightarrow J(j)$.

Note that Landman's formal requirements are too strong: they result in a trivial or contradictory theory.

Theorem: *Landman's Axioms allow all individuals to have all predicates hold of them.*
Proof: Consider any individual i and property P. Axiom (ii) $(i\,\varrho\,P)(P)$ provides the antecedent for Axiom (viii) $(i\,\varrho\,P)(Q) \rightarrow P(i)$. An application of *modus ponens* thus yields $P(i)$. □

This suggests that the axioms should only consider an individual under a guise if the individual has that guise as a property. That is, we should only consider the expression $(i\,\varrho\,p)$ as a restricted individual if p holds of i. Although not captured by Landman's axioms, this does seem to be his intention. This is apparent when he makes some comments on the relatedness of the guise under which an individual is restricted, to the properties predicated of the restricted individual.[3] Of particular relevance to the above theorem, he says that some objects have fewer aspects than others, *cards* having fewer than *humans*, for example. This would suggest that Landman only wants his axioms to consider the expression $(i\,\varrho\,p)$ if $p(i)$.

Four of Landman's axioms are concerned with overcoming the strong typing of the representation, and just give 'predication' of individuals under guises the expected behaviour. In his initial system, only *singular* individuals can be restricted. If *plural* individuals are considered, then both the theory of plurals, and the theory of intensional individuals need to be strengthened. For these reasons, he suggests that the theory may be more appropriately treated in Turner's weakly typed property theory (Turner 1992). In addition, he notes that individuals under guises could be represented as terms, rather than generalised quantifiers. This is the approach adopted in my initial property-theoretic treatment.

3 Property Theory

I shall now present a version of property theory PT, Turner's axiomatisation of Aczel's Frege Structures (Turner 1992; Aczel 1980). This is a language with a highly intensional notion of properties and propositions which avoids the paradoxes without banning self-predication through strong typing. In PT, propositions, and properties, are taken to be primitive. A property like "red" is not the set of red things, it is just itself, the property of being red. Similarly, the proposition, "$2 + 2 = 4$" is not merely a truth value, or a set of possible worlds, but is, instead, a basic object, different from "$e^{i\pi} + 1 = 0$", even though, from the laws of mathematics, these propositions must always be true together.

3. For example, being well paid, and a judge, both relate to jobs. Landman contends that this gives rise to greater intensionality in the subject of propositions like "John-as-a-judge is well paid". He believes that some aspects, or guises, are naturally more intentional than others, *judge* is more intensional than *drunk*, which is in turn more intensional than *man with a big nose*.

3.1 General Framework

Conceptually, PT can be split into two components, or levels. The first is a language of terms, which consists of the untyped λ-calculus, embellished with logical constants. A restricted class of these terms will correspond to *propositions*. When combined appropriately using the logical constants, other propositions result. As an example, given the propositions t, s, the 'conjunction' of these, $t \wedge s$, is also a proposition, where $\wedge$ is a logical constant.

Some of the propositions will, further, be *true* propositions. When combining propositions with the logical constants, the truth of the resultant proposition will depend upon the truth of the constituent propositions. Considering the previous example, if t, s are both propositions, then $t \wedge s$ will be a true proposition if and only if t and s are true propositions.

There may be terms that form propositions when applied to another term. These terms are the properties. The act of predication is modelled by λ-application.

The essential point to note is that this is a highly intensional theory as the notion of equality is that of the λ-calculus: propositions are not to be equated just because they are always true together; similarly, properties are not to be equated just because they hold of the same terms (i.e. form true propositions with the same terms).

There are problems with the theory so far: the logical constants have no proof theory; and the notions of being a proposition, or a true proposition, cannot be expressed within this language of terms. That is, although we can consider terms as propositions, or true propositions, and comprehend how the proposition-hood and truth of a term depends upon the proposition-hood and truth of its constituent terms, we cannot express these notions formally *within* the language of terms: some *meta-language* is required. This is the purpose of the second component of PT: the language of *well formed formulae* (wff). This is a first-order language where the terms (the objects which can be quantified over) are those of the λ-calculus extended with logical constants, as discussed above. The language of wff has two predicates, P for 'is a proposition', and T for 'is a true proposition'. Clearly, this gives the formal means for axiomatising the behaviour of propositions and true propositions. For example, the informal discussion concerning the behaviour of the logical constant $\wedge$ can be formalised as follows:

> "given the propositions t, s, the conjunction of these $t \wedge s$ is also a proposition":
>
> $$\mathrm{P}(t) \,\&\, \mathrm{P}(s) \rightarrow \mathrm{P}(t \wedge s)$$
>
> "if t, s are both propositions, then $t \wedge s$ will be a true proposition if and only if t and s are true propositions":
>
> $$\mathrm{P}(t) \,\&\, \mathrm{P}(s) \rightarrow (\mathrm{T}(t \wedge s) \leftrightarrow (\mathrm{T}(t) \,\&\, \mathrm{T}(s)))$$

Axioms concerning T must be restricted so that only terms that are propositions are considered.

The distinction between wff and terms can be taken to be akin to that between extension and intension in Montague semantics (Dowty et al. 1981). In that theory, however, intensions are derived from extensions. As a consequence, the equality of intensions is that of the extensions, so propositions will be equated if they are always true together, and properties will be equated if they hold of the same objects. This is in contrast to PT, where the intensions are basic. Propositions in the language of terms may have the same truth conditions when T is applied, but this does not force them to be the same proposition, so we might have:

$$\mathrm{T}(s) \leftrightarrow \mathrm{T}(t)$$

but that does not mean that the terms are equal:

$$s = t$$

Similarly, in the language of wff, properties may hold of the same terms, yet they may be distinct. The λ-equality of terms is thus weaker than the notion of logical equivalence obtained when considering truth conditions in the meta-language.

It can be seen that PT characterises a Frege Structure: two classes of λ-terms are defined by P and T as below:

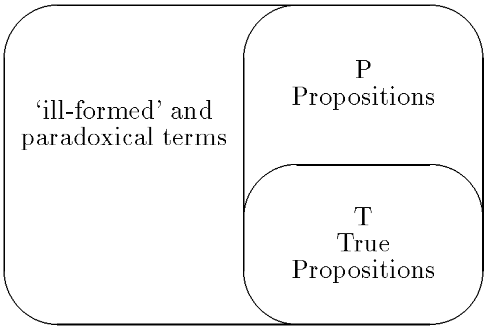

Diagram: A Frege Structure

3.2 Formal Theory

The following presents a formalisation of the languages of terms and wff, together with the axioms that provide the closure conditions for P and T.

The Language of terms

Basic Vocabulary:

Individual variables:	$x, y, z, \ldots$
Individual constants:	$c, d, e, \ldots$
Logical constants:	$\vee, \wedge, \neg, \Rightarrow, \Xi, \Theta$

Inductive Definition of Terms:

(i) Every variable or constant is a term.
(ii) If t is a term and x is a variable then $\lambda x.t$ is a term.
(iii) If t and t' are terms then $t(t')$ is a term.

The Language of Wff

Inductive Definition of Wff:

(i) If t and s are terms then $s = t, \mathrm{P}(t), \mathrm{T}(t)$ are atomic wff.
(ii) If φ and φ' are wff then $\varphi \,\&\, \varphi', \varphi \vee \varphi', \varphi \rightarrow \varphi', \sim\varphi$ are wff.
(iii) If φ is a wff and x a variable then $\exists x \varphi$ and $\forall x \varphi$ are wff.

The theory is governed by the following axioms:

Axioms of The $\lambda\beta$-Calculus

$$\begin{aligned} \lambda x.t &= \lambda y.t[y/x] \; y \text{ not free in } t \\ (\lambda x.t)t' &= t[t'/x] \end{aligned}$$

This defines the equivalence of terms.

The closure conditions for proposition-hood are given by the following axioms:

Axioms of Propositions

(i) $\mathrm{P}(t) \,\&\, \mathrm{P}(s) \rightarrow \mathrm{P}(t \wedge s)$
(ii) $\mathrm{P}(t) \,\&\, \mathrm{P}(s) \rightarrow \mathrm{P}(t \vee s)$
(iii) $\mathrm{P}(t) \,\&\, \mathrm{P}(s) \rightarrow \mathrm{P}(t \Rightarrow s)$
(iv) $\mathrm{P}(t) \rightarrow \mathrm{P}(\neg t)$
(v) $\forall x \mathrm{P}(t) \rightarrow \mathrm{P}(\Theta \lambda x.t)$
(vi) $\forall x \mathrm{P}(t) \rightarrow \mathrm{P}(\Xi \lambda x.t)$
(vii) $\mathrm{P}(s \approx t)$

Truth conditions can be given for those terms that are propositions:

Axioms of Truth

(i) $\mathrm{P}(t) \,\&\, \mathrm{P}(s) \rightarrow (\mathrm{T}(t \wedge s) \leftrightarrow \mathrm{T}(t) \,\&\, \mathrm{T}(s))$
(ii) $\mathrm{P}(t) \,\&\, \mathrm{P}(s) \rightarrow (\mathrm{T}(t \vee s) \leftrightarrow \mathrm{T}(t) \,\mathrm{v}\, \mathrm{T}(s))$
(iii) $\mathrm{P}(t) \,\&\, \mathrm{P}(s) \rightarrow (\mathrm{T}(t \Rightarrow s) \leftrightarrow \mathrm{T}(t) \rightarrow \mathrm{T}(s))$
(iv) $\mathrm{P}(t) \rightarrow (\mathrm{T}(\neg t) \leftrightarrow \sim \mathrm{T}(t))$
(v) $\forall x \mathrm{P}(t) \rightarrow (\mathrm{T}(\Theta \lambda x.t) \leftrightarrow \forall x \mathrm{T}(t))$
(vi) $\forall x \mathrm{P}(t) \rightarrow (\mathrm{T}(\Xi \lambda x.t) \leftrightarrow \exists x \mathrm{T}(t))$
(vii) $\mathrm{T}(t \approx s) \leftrightarrow t = s$
(viii) $\mathrm{T}(t) \rightarrow \mathrm{P}(t)$

The last axiom states that only propositions may have truth conditions.

Note that the quantified propositions $\Theta\lambda x.t$, $\Xi\lambda x.t$ can be written as $\Theta x(t)$, $\Xi x(t)$, where the λ-abstraction is implicit.

The notions of n-place relations can be defined recursively:

(i) $Rel_0(t) \leftrightarrow \mathrm{P}(t)$
(ii) $Rel_n(\lambda x.t) \leftrightarrow Rel_{n-1}(t)$

We can write $Rel_1(t)$ as $\mathrm{Pty}(t)$.

3.3 NL Examples

As an example of how PT can be used in the semantics of natural language, the sentence:

Every boy laughed.

could be represented as a term, as follows:

$$\Theta x(\mathrm{boy}' x \wedge \mathrm{laughed}' x)$$

This object is independent of any truth conditions. To find the truth conditions of the sentence, we must first show that the term representing it is a proposition, that is:

$$\mathrm{P}(\Theta x(\mathrm{boy}' x \wedge \mathrm{laughed}' x))$$

This is an expression in the language of wff. According to the axioms for P, this will hold if:

$$\forall x(\mathrm{P}(\mathrm{boy}' x) \,\&\, \mathrm{P}(\mathrm{laughed}' x))$$

If the sentence is a proposition, then its truth conditions are given by:

$$\mathrm{T}(\Theta x(\mathrm{boy}' x \wedge \mathrm{laughed}' x))$$

According to the axioms, this holds if and only if:

$$\forall x(\mathrm{T}(\mathrm{boy}' x) \,\&\, \mathrm{T}(\mathrm{laughed}' x))$$

Not all sentences will express propositions. As an example, the axioms should not allow the representation of:

This sentence is false.

to be a proposition, otherwise the theory would fall foul of the paradoxes.

Not all logical constants in the representations of sentences will be interpreted as logical connectives in the truth conditions. The sentence:

Mary believes that every boy laughed.

might be represented with the term:

$$\text{believe}'(\Theta x(\text{boy}'x \wedge \text{laughed}'x))\text{mary}'$$

If this is a proposition, then in its truth conditions T will not apply to "every boy laughed":

$$\text{T}(\text{believe}'(\Theta x(\text{boy}'x \wedge \text{laughed}'x))\text{mary}')$$

This corresponds to the idea that the object of a believe is an intensional proposition, not a truth value, or set of possible worlds.

4 Roles in PT

Now I shall embody Landman's ideas in PT. The axioms are weakened in a manner implicit in the text of his paper. This results in a logic similar to Landman's but without the complications of types, where all individuals are terms. Those of Landman's axioms which are used to overcome strong typing correspond with theorems of this PT formalisation. Any structure imposed on the terms, such as a Boolean algebra-like structure used in the semantics of plurals, can be imposed uniformly on all terms without strengthening the theory.

4.1 Informally

We must limit those objects considered to be individuals-under-a-guise (restricted terms). Presumably, a term may only be considered under the guise of having a property if it has that property. It is not desirable to restrict the formation of terms themselves, as this completely changes the character of the theory. Those terms considered as restricted terms must instead be limited, just as those terms considered as propositions and properties are limited, without restricting the language of terms. We can define a new predicate $\mathcal{RT}$ in the language of wff which characterises restricted terms. A term of the form $i\,\varrho\, p$ (i under the guise of being p), will only be in $\mathcal{RT}$ if p is a property which holds of i:

$$\mathcal{RT}(i\,\varrho\,p) \leftrightarrow (\text{Pty}(p)\,\&\,\text{T}(pi))$$

Via the definition of Pty, this is provably equivalent to:

$$\mathcal{RT}(i\,\varrho\,p) \leftrightarrow \text{T}(pi)$$

Additionally, we can add an operator Ext to the language of terms. This will return the underlying individual in a restricted term. We can also give an 'internal' form, $\widehat{rt}$, of the predicate $\mathcal{RT}$, for use in intensional contexts.

4.2 Formally

The basic vocabulary of the language of terms is extended with:

$$\varrho\,,\mathit{Ext},\widehat{rt},(\oplus,\sigma)$$

The theory is governed by the axioms of the $\lambda\beta$-Calculus as before. The language of wff is also as before, perhaps with further axioms for the plural constants $\oplus, \sigma$, which can be used to represent plural sum, and supremum respectively (Fox 1993).

We can define $\mathcal{RT}$ such that is satisfies $\mathcal{RT}(i\,\varrho\,p) \leftrightarrow \mathrm{T}(pi)$ with:

Definition 1 Restricted terms:

$$\mathcal{RT}(x) =_{def} \exists pi(\mathrm{Pty}(p) \,\&\, (i\,\varrho\,p) = x \,\&\, \mathrm{T}(pi))$$

The intuition behind Landman's Axiom (viii) is embodied in this definition: if we can consider the restricted term of "John under the aspect of being a judge", then John is a judge.

The term $\widehat{rt}$, the internal analogue of $\mathcal{RT}$, can be given the following truth conditions:

Axiom 1 'Internalised' $\mathcal{RT}$:

$$\mathrm{Pty}(p) \rightarrow (\mathrm{T}(\widehat{rt}(t\,\varrho\,p)) \leftrightarrow \mathcal{RT}(t\,\varrho\,p))$$

Landman's Axiom (i) says "John-as-a-judge is a judge". In PT we first must preface this with the restriction that "John-as-a-judge" is a legitimate restricted term:

Axiom 2 "*i*-as-a-*p*", is a *p* (cf. Landman's Axiom (i)):

$$\mathcal{RT}(i\,\varrho\,p) \rightarrow \mathrm{T}(p(i\,\varrho\,p))$$

In Landman's theory "John-as-a-judge" has the property of being equal to "John" (Axiom (ii)). In PT this is of little use: to say that "John-as-a-judge" is "John" amounts to loosing the distinction between terms and restricted terms. It is better, in this formalisation, to say that the *underlying individual* of "John-as-a-judge" is "John":

Axiom 3 The 'extension' of "*i*-as-a-*p*" is *i* (cf. Landman's Axiom (ii)):

$$\mathcal{RT}(i\,\varrho\,p) \rightarrow \mathit{Ext}(i\,\varrho\,p) = i$$

Axiom (iii) of Landman's theory says "John-as-John" is the individual sublimation of John. I think this is because he takes restricted individuals to be quantifiers (which are of the same type as individual sublimations). When restricted individuals are taken to be terms, it seems more intuitive to have "John-as-John" being "John" himself, rather than the individual sublimation of John.

Axiom 4 "*i*-as-*i*" is *i* (cf. Landman's Axiom (iii)):

$$i\,\varrho\,(\lambda x(x \approx i)) = i$$

We naturally have guises as proper filters of properties, as an individual-under-a-guise is just a term. Because of the strong typing of Landman's base theory, and because he does not take individuals under guises to be terms, he can only obtain this behaviour via additional axioms. Further, when plurals are added to Landman's theory he requires two plural structures: one structure for ordinary individuals; and a second for individuals restricted to particular guises. He also needs type shifting for predicates—to allow them to apply to individuals from both structures—and requires additions to his theory of restricted individuals to cope with plural individuals under guises. In the property theoretic treatment, only one plural structure is required over the language of terms, and the theory can represent plural individuals under guises with no alterations (Fox 1993).

4.3 A Problem

As PT is weakly typed, and this formalisation treats both ordinary individuals and individuals-under-guises as terms, we might question the limits to be placed on the kinds of objects which can be restricted. For example: can restricted terms themselves be restricted? The term:

$$(j \varrho J) \varrho H$$

exists, according to the weak syntax of the language of terms, but should it be in $\mathcal{RT}$? It cannot just be made equivalent to:

$$j \varrho \lambda x(Jx \wedge Hx)$$

because the existence of the former implies $\mathrm{T}(H(j \varrho J))$, whilst the latter would imply $\mathrm{T}(Hj)$.

Taken in combination with a theory of conjoined terms, we may enquire whether sums of restricted terms themselves can be restricted:

$$((j \varrho J) \oplus (b \varrho H)) \varrho S$$

where $((j \varrho J) \oplus (b \varrho H))$ is the plural sum of $((j \varrho J)$ and $(b \varrho H))$.

It is possible to have axioms that are too weak to decide these issues, or to assume that they never arise in the consideration of natural semantics. However, rather than pursue the question of iterated constructions here, I shall promote an alternative analysis of the examples using *property modifiers*, where the appropriate behaviour can be obtained by considering theories of NL modifiers, such as adjectives and adverbs. This will bring into question whether we need to take individuals-under-guises as a new basic category in the ontology of natural language semantics.

5 Property Modifiers

Landman notes that in many cases, with unary predication of a term, the appropriate role can equivalently be taken to modify the act of predication, rather than the term (Landman 1987). Instead of representing the 'strict judge' example as before:

> John-as-a-judge is strict.
> John-as-himself is liberal.

a *property modifier* approach would effectively represent this as:

> John is a strict judge.
> John is liberal. (or 'John is a liberal person.')

This has some advantages. Property modifiers are already required in the semantics of natural language to represent modifier expressions such as adjectives and adverbs. With this representation, roles can share the axioms and behaviours of such modifiers. A property modifier analysis of Landman's examples thus uses existing notions, rather than requiring a new category of terms. Also, demonstratives of the same kind do not have to denote different individuals depending upon the relevant role: it is the act of predication that gives rise to intensionality. Thus the demonstratives in the following sentences:

> That is the Morning Star.
> That is the Evening Star.

may denote the same individual, although subsequent predication of the individual will be modified as appropriate.

Landman argues against using a property modifier analysis in general, because of comparatives such as:

> The judge and the cleaner have different incomes.

where the same individual may be both judge and cleaner. As the nominals give rise to the roles, there are the same number of roles as there are nominals. If each nominal gives rise to a mention of an individual in the representation, then there will be the same number of roles as mentions of individuals. Thus the representation of this sentence is straightforward if the roles modify individuals. In contrast, according to Landman, the roles cannot give rise to property, or predicate modifiers, as there are two roles but only one modifiable act of predication in this example.[4]

This assumes that in the truth condition, the application of the verb phrase 'have different incomes' to the subject noun phrase can only be considered as giving rise to a single irreducible act of predication. However, against this, and against Landman's counter example, there is nothing to prevent the appropriate truth conditions giving rise to several acts of predication. In particular, there is nothing to prevent there being an equal number of predications as roles, or constituent nominals. This can be seen if the comparative is paraphrased as:

> The income *from cleaning* earned by the person who is the cleaner is different to the income *from judging* earned by the person who is the judge.

If John is both cleaner and judge, this becomes:

> John's income as a judge is different to his income as a cleaner.

The truth conditions of the original sentence can be represented as:

$$\begin{aligned}\mathrm{T}(\Xi ab(\,&\mathrm{income}'a \wedge \mathrm{income}'b \wedge \\ &((\mathrm{earn}'a)\mathrm{judge}')[\![\text{The judge}]\!] \wedge \\ &((\mathrm{earn}'b)\mathrm{cleaner}')[\![\text{The cleaner}]\!] \wedge \\ &\mathrm{different}'(a \oplus b)))\end{aligned}$$

and if John is both judge and cleaner, this is equivalent to:

$$\begin{aligned}\mathrm{T}(\Xi ab(\,&\mathrm{income}'a \wedge \mathrm{income}'b \wedge \\ &((\mathrm{earn}'a)\mathrm{judge}')\mathrm{John}' \wedge \\ &((\mathrm{earn}'b)\mathrm{cleaner}')\mathrm{John}' \wedge \\ &\mathrm{different}'(a \oplus b)))\end{aligned}$$

The property $\mathrm{different}'$ takes a sum for its argument to ensure a consistent interpretation of conjunction, and to allow arbitrary numbers of terms to be "different". Some meaning postulate such as:

$$\mathrm{T}(\mathrm{different}'(a \oplus b)) \leftrightarrow a \neq b$$

would have the desired outcome.[5]

---

4. Note that a role may either modify the main predicate in the verb phrase, or, equivalently, it may be the main predicate, modified by the verb phrase. I assume the latter in my representations.
5. This could be generalised to arbitrary nominal conjunctions, although it is not clear to me that we can, or should, attempt to completely decompose the meaning of "different" in the general case.

Thus, all of Landman's examples can be analysed without recourse to a new category of intensional individuals.

Note that it is possible to define restricted equivalences between a property modifier representation of the examples, and a representation which uses individuals under guises. If we restrict ourselves to truth conditions which give at least one predication relation for each role, and we do not iterate roles, then the following provides equivalences between the representations:

From Property Modifiers to Restricted Terms: If we rewrite strict$'$, when acting as a property modifier, as $\lambda p.p(\text{strict}')$, and judge$'$, when acting as a property, as $\lambda q \lambda x.q(x\,\varrho\,\text{judge}')$ then property modifier expressions are effectively rewritten as expressions involving individuals under guises. That is, strict$'$judge$'(x)$ becomes strict$'(x\,\varrho\,\text{judge}')$.

From Restricted Terms to Property Modifiers: Assuming that postfix $\varrho\,p$ is rewritten as a prefix, and we rewrite the property strict$'$ as $\lambda p.p(\text{strict}')$, and the term ϱ is defined to be $\lambda q \lambda x \lambda p(pqx)$ then propositions given in terms of individuals under guises are equivalent to propositions involving property modifiers. That is, strict$'(\,\varrho\,\text{judge}'x)$ becomes strict$'$judge$'(x)$.

Thus, Landman's intensional individuals can be taken to be definable in terms of more familiar notions.

Some issues have not been addressed here, for example: (1) the question of where appropriate guises come from; (2) the nature of anaphoric reference to 'individuals-under-a-guise' when represented with property modifiers; and (3) how we might systematically obtain the appropriate representations of the examples (especially those involving comparatives).

Regarding (1) *the problem of where appropriate guises might come from*: if we have the two sentences:

> John is tall.
> Everest is tall.

we really mean something along the following lines:

> John is a tall *person*.
> Everest is a tall *mountain*.

yet the appropriate roles are not contained within the sentence. It may be that the appropriate guise can be obtained from some sortal hierarchy.

With respect to (2) *the nature of anaphoric reference*: from the sentences:

> There is [a judge]$_i$.
> [She]$_i$ is strict.

we might wish to conclude:

> There is a strict *judge*.

or, more explicitly:

> There is someone who is a strict *judge*.

It may not be apparent how we can do this if we stick rigidly to a property modifier representation. However, it should be pointed out that the argument in favour of property modifiers presented here is intended to show that we do not require a new basic category of terms. That is, all sentences apparently involving individuals under a guise can be treated by alternative means. This should not be taken as an argument against using syntactic sugar—where the guise is carried by the individual—when appropriate.

Finally, concerning (3) *systematically obtaining appropriate representations of the examples*: I shall not attempt to illustrate a reductive compositional analysis of a sentence to some term involving logical constants, with their fixed truth-conditional behaviour (although such an analysis is no doubt possible). Instead, I propose to compositionally produce some term which is neutral with respect to the appropriate guises (and to scoping). Additional axioms can be added to obtain the various truth conditions of such neutral terms (Fox 1993). This is in the spirit of PT: in the same way that it may be inappropriate to reduce the notion of *property* to that of *set*, it may also be inappropriate to reduce a given natural language construction to some particular representation involving logical constants, with their fixed interpretation in the truth-conditions.

6 Conclusions

I have shown that Property Theory allows us to consistently formalise Landman's ideas. This results in a simpler theory. Some of Landman's axioms become theorems. As all individuals are taken to be terms, the theory is compatible with treatments of plurals without additional complications for either the theory of plurals or that of individuals under guises. All that is required is to add some Boolean algebra-like structure to a class of terms which includes the individuals under guises.

Further, I have indicated how all of Landman's examples can be treated using property modifiers. This shows that there is no need to take these individuals under guises as some new category of terms. A property modifier treatment of the examples uses existing notions—property modifiers are probably already required for the semantics of modifiers such as adjectives and adverbs—and thus avoids any problematic intuitions concerning roles and guises. The weak typing and the intensionality of PT provides a suitable framework for their representation.

This does not mean that the concept of an individual acting under a guise should not be represented. Indeed, as I have shown, it is possible to define an individual-under-a-guise in terms of property modifiers, and *visa versa*. But it does mean that such a notation can be regarded as syntactic sugar, rather than an argument for re-appraising the ontology required for the semantics of natural language.

References

Aczel, P.: 1980, Frege structures and the notions of proposition, truth and set, in Barwise, Keisler, and Keenan (eds.), *The Kleene Symposium*, North Holland Studies in Logic, pp 31–39, North Holland

Dowty, D., Wall, R., and Peters, S.: 1981, *Introduction to Montague Semantics*, Reidel, Dordrecht

Fox, C.: 1993, *Plurals and Mass Terms in Property Theory*, Ph.D. thesis, University of Essex, Colchester, U.K.

Landman, F.: 1987, Groups, plural individuals and intentionality, in J. Groenendijk, M. Stokhof, and F. Veltman (eds.), *Proceedings of the Sixth Amsterdam Colloquium*, pp 197–217, ITLI, University of Amsterdam

Landman, F.: 1989, 'Groups', Part I, II, *Linguistics and Philosophy* 12, 559–605, 723–744

Thomason, R.: 1980, A modeltheory for propositional attitudes, *Linguistics and Philosophy* 4

Turner, R.: 1992, Properties, propositions and semantic theory, in M. Rosner and R. Johnson (eds.), *Computational Linguistics and Formal Semantics*, Studies in Natural Language Processing, pp 159–180, Cambridge University Press, Cambridge