Existence and Freedom

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Gedenkschrift · Goldsmiths College · 2nd September 2016

Outline

Variables in Programming Languages

- What are the values of program variables?
- Potential values constrained by their type (and representation)
- Actual value determined by the execution path

```
1.  int x
2.  ...
3.  if C
4.  then x ← A
5.  else x ← B
6.  ...
```

- Value of `x` changes, and depends on execution path

Abstract values of variables

- In general, at a given point a variable might not be assigned a value
- Also a variable may have a value, but we not know what it is
- We can use abstract values to represent these cases
  - e.g. $\bot$ and $\top$ (respectively)

Abstract values: example

- Consider:

```
1.  int x
2.  ...
3.  if C
4.  then x ← A
5.  else x ← B
6.  ...
```

  - At `2.`, `x` is undefined ($\perp$)
  - At `6.`, `x` is "over"-defined ($\top$)
- We can use a test $x\downarrow$ to determine whether a variable x has a concrete (non-abstract) value
  - E.g. at `2.`, `x`$\downarrow$ is false, from `4.` onward, `x`$\downarrow$ is true

Programming languages and undefined values

- Some languages and types support explicit "undefined" values, and operations on them
  - E.g. `null` in SQL, and `NaN` in IEEE floats
- Most do not

Meta-languages and undefined values

- "Meta-languages" can be used to express and formalise properties of programs
- They need to allow variables to have such undefined values even when they are not represented (or representable) in the object programming language
- Now $x\downarrow$ means the value of x has a concrete value in the programming language[1]

[1] There are other options; we could take it to mean something like "the value of x is computable" — cf. conditioned slicing

Issues in formalisation

- If we want to quantify over the values of variables, should the domain of quantification include these abstract values?
- There are two options, which we shall call the
  1. *partial* and
  2. *free*

  approaches[2]

[2] In practice, the term "free" is often applied to both approaches

Partial approach — internal domains

- On the partial approach, we only quantify over the concrete/defined values, for which $\cdot\downarrow$ holds
- The quantifier elimination rules need to impose a "side-condition" that the value to be substituted is defined, e.g.:

$$\frac{\forall x \cdot \varphi \quad a\downarrow}{\varphi[x := a]}\ \forall -$$

- This is like quantifying over the "internal" range of concrete values overtly supported by the programming language

Free approach — external domains

- On the free approach, we quantify over both concrete/defined values and abstract values
- When dealing with concrete behaviour, any constraints on defined-ness are expressed within the scope of the quantifier, rather than as a side-condition, e.g.:

$$\frac{\forall x \cdot x\downarrow \to \varphi}{(x\downarrow \to \varphi)[x := a]}$$

- This is like classical quantification, but over the "external" range of values required by the meta-level analysis of programs

Free and partial logic

- Both *partial* logic and *free* logic accommodate $\perp$
  1. Partial logic allows terms to have undefined values, but the (partial) quantifiers only range over defined values
  2. Free logic allows terms to have undefined values, over which the (free) quantifiers can range[3]
- We could allow the notion of internal and external domain to include other "abstract" values, such as $\top$ ... but we won't pursue this here
- If we wish to use both quantifiers together, then we can write $\exists\!\downarrow$ for partial quantification, and $\exists$ for free quantification

[3] In the wider literature, the term "free logic" is used to refer to range of logics, including what we describe as partial logic

From computer science to philosophy

- The notion of defined-ness, and the different ways of formulating quantification in the face of undefined values (such as $\perp$) are of relevance in the analysis of existence in analytic philosophy
- The term "free logic" comes from philosophy — meaning 'free from ontological commitments'[4]
- Free logic arises in counter-arguments to the position of Quine and others that formal existence in classical logic implies real existence in the world

[4] "logic free of existence assumptions with respect to its terms, singular and general" (Karel Lambert, 1960)

Nature of entities and stuff

- We focus on on accounts of ontology based on mereological notions
- These have been advocated by D Lewis and T Sider, among others
- They can be seen to be motivated in part by arguments made against ontological pluralism[5]
- Some of the arguments are also based on particular views of the nature of logical, and existential quantification in particular

[5] The concern being to resist an "anything goes" ontology

Mereology

Mereology is a theory of parts and wholes

- Every physical thing can be defined in terms of parts and wholes
- If we have two "things" a, b, then their "fusion" $(a \oplus b)$ is also a "thing"
- Can be formalised (Leonard and Goodman)
  - E.g. $\forall xy \cdot \exists a = x \oplus y$

Everything is either a "simple" (monad) or a fusion

Mereology background

- Grew out of *nominalism*
- Formally it resembles lattice theory, but is from a tradition that rejects sets and set theory
- Open questions as to whether theory should be atomic, and the precise status of macro-objects
- Only one of many theories about the nature of physical being

Universalism

- In this context, *(mereological) universalism* is the view that any two (or more) entities can be considered together as a *fusion* that then constitutes a new object
- This may be motivated by arguing that once we allow things to be fused, then there are no principled reasons for restricting this notion
  - One argument is that any such restriction would then require a restricted or vague notion of existence
  - Claims are made that any such restricted notion of existence is incoherent

Universalist Nihilism

- A further argument that is made (e.g. by Sider) is that the *only* things that exist are fusions (and simples)
- Given a fusion of a and b, there is nothing beyond that object than the thing that is defined by its composition out of the constituent parts a and b (i.e. $a \oplus b$)
- We call this "universalist nihilism" to distinguish it from (mereological) nihilism, where *nothing* combines; there is no entity corresponding to the fusion of two primitive things

Issues

- Mereological universalism is not free from criticism
- It can seem both too exclusive and too promiscuous

Brooms and rivers

- A simple-minded reduction of objects to physical stuff breaks down in everyday understanding
  - The broom with the replaced handle, and the replaced head (cf the ship of Theseus)
  - The flowing river — that's never the same twice (cf Heraclitus)
- There seem to be notions of continuity, and identity, that go beyond the physical

The universalists' response

- The broom/river is captured by some mereological "space-time envelope"
- So nothing more than mereology is required (according Lewis & Sider)[6]

[6] But what then determines/identifies the relevant space-time envelopes?

Promiscuity

- Universalism is too promiscuous in claiming that something corresponds to a (natural) object (cf Kathrin Koslicki)
- It also seems unnatural: where are the houses, trees etc.?
  - (Related to previous concerns about identity and continuity)

The universalists' response

- The alternative (ontological pluralism?) is worse

The intermediate position

An alternative to universalism and (non-mereological) nihilism:

- Some fusions of things constitute natural objects, others do not

Vagueness and Existence

- This motivates some to argue that "existence" is in some sense vague: it is vague whether a "exists", where a is the fusion of two (or more) objects
- Much discussion is motivated by this ...
  ... which we will skip[7]

[7] Relevant, but potentially a little bit of a red herring?

Separation of concerns

- We can allow all fusions to "exist", while denying they then necessarily constitute "natural" objects
- Cf abstract vs. concrete values of programme variables
- We can use free/partial logic to separate out these concerns
  - We can use $a\downarrow$ to mean a fusion corresponds to a "normal" object
  - The exists of a fusion f does not entail any correspondence to a normal object $(f\downarrow)$

Classical Logic for universalism

- We can use quantification from our "free logic" for mereological universalism

$$\forall xy \cdot \exists a = x \oplus y$$

- But this does not mean $a\downarrow$, as the quantifier ranges over both 'defined' and 'undefined' objects, without distinction

Partial Logic for normal objects

- We can use partial quantification $(\exists\downarrow)$ for "normal" objects

$$\frac{\exists\downarrow x \cdot x = a}{a\downarrow}$$

Ontological consequences

- Universalists and those adopting the intermediate position differ only(!) on the quantificational domain they consider relevant for ontological questions
- Essentially this allows universalists to have their mereological cake while the non-universalists can eat it
- From

$$\forall xy \cdot \exists a \cdot a = x \oplus y$$

we cannot infer $\exists \downarrow a \cdot a = x \oplus y$

Outstanding concerns

- The forgoing addresses some concerns but...
- Such an approach is still excessively reductive (equating objects with stuff)

. . . *Brooms and rivers revisited*

- Recall: universalists would say that what we consider as "being" the broom/river can still be captured (e.g. as some mereological space-time envelope)
- But don't we still need some independent conception of what is being characterised to identify appropriate "space-time envelopes"?
  - also where vagueness creeps in
  - (Universalists will say that is "just" linguistics. . .)

A further move

- We can introduce a notion of a natural/linguistic object's (i) mereological extension (εi)
  - we can have "the broom" (b), and distinguish it from what it physically corresponds to in the world (εb)
- We can then interpret $a\downarrow$ as *a being the extension (or manifestation) of some (linguistic) object*, i.e. $a\downarrow \triangleq \exists x \cdot \varepsilon x = a$
- We can go on to consider the behaviours and interactions of $\downarrow$, ε and $\oplus$ etc.[8]

[8] It is the notion of an extension (ε) that can be considered vague, rather than existence itself

Intensional individuals

- Crudely put, this can be seen to distinguish between
  1. the intensional, subjective realm of language and experience
  2. the (reductive) physical realm
- A (crude) parallel in program analysis would be the distinction between
  1. abstract data types (the abstract, "linguistic" realm of program comprehension)
  2. concrete representations and operations (the physical implementation)

Counter arguments

There are counter arguments to this kind of proposal

- Those that regard classical quantification as somehow "natural" would object to free logic, in favour of universalism, and against "structural universals" (cf 'abstract data types')
- Essentially some such arguments appear to beg the question — although in subtle ways — and resemble faith more than reason
- "That the objects in the domain [of quantification] have or lack any particular ontological status is a philosophical interpretation of the formal semantics" (John Nolt)

Summary

We have sketched how notions of free logic and "defined-ness" relevant for theoretical computer science can be applied to consider questions in analytic philosophy

- Rather than deny universalism, or insist on vague quantification, the idea has been to show how universalism might fit within a larger framework
- On this account, disagreements about existence then appear to be differences in judgements about which domain counts as the most "natural"

Conclusions

- Considerations of the "ontology" of languages and formalisation of abstractions in computer science helps give insights into more general philosophical questions

THE END